Random shearing by zonal flows and transport reduction

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The physics of random shearing by zonal flows and the consequent reduction of scalar field transport are studied. In contrast to mean shear flows, zonal flows have a finite autocorrelation time and can exhibit complex spatial structure. A random zonal flow with a finite correlation time τ_{ZF} decorrelates two nearby fluid elements less efficiently than a mean shear flow does. The decorrelation time is $\tau_D=(\tau_\eta/\tau_{ZF}\Omega_{rms}^2)^{1/2}$ (τ_η is the turbulent scattering time, and Ω_{rms} is the rms shear), leading to larger scalar field amplitude with a slightly different scaling ($\propto\tau_D/\Omega_{rms}$), as compared to the case of coherent shearing. In the strong shear limit, the flux scales as $\propto\Omega_{rms}^{-1}$. © *2004 American Institute of Physics*. [DOI: 10.1063/1.1808455]

Shear flows are ubiquitous in a variety of physical systems, including differential rotation in galaxies and stars,[1] zonal flows in major planets,[2] laboratory plasmas,[3] and earth atmosphere.[4] These coherent structures play a distinctive role in determining transport in plasmas due to the dramatic effect of shearing on regulating turbulence (see, e.g., Ref. 3). The reduction of transport results from the change not only in the turbulence intensity but also in the correlation time and cross phase. For instance, in the case of a passive scalar field χ,[5] the transport is reduced as $\langle\chi v_x\rangle\propto\Omega^{-1}$ by a stationary linear shear flow $\mathbf{U}=x\Omega\hat{y}$, mainly because the turbulence amplitude decreases as $\langle\chi^2\rangle\propto\Omega^{-5/3}$, with no significant change in the cross phase ($\langle\chi v_x\rangle/\sqrt{\langle\chi^2\rangle\langle v_x^2\rangle}\propto\Omega^{-1/6}$) (cf. Ref. 6). When the turbulent flow v_x evolves self-consistently, its amplitude is also reduced by shearing, resulting in a stronger reduction in the transport.[7]

Zonal shear flows, often encountered in a variety of systems, are often self-generated by the underlying turbulence via Reynolds stress,[8] and thus very likely to be structured and possibly even random in both space and time, on account of the broad range of their excitation via modulational instability. Thus, in contrast to smooth, static mean flows, the zonal flow patterns can be expected to have finite correlation time and complex spatial structure. These shear flows, which are nonlinearly driven by turbulence, are so-called zonal flows and should be distinguished from mean flows. Zonal flows, for example, are shown to play a crucial role in regulating turbulence.[9] Therefore, *it is important to understand how much transport is reduced,* in general, *by shear flows with finite correlation time* τ_{ZF} (Ref. 10) *and complex spatial form. Nearly all of the previous work on shear flow regulation of transport has considered the case of the mean shear, only*. In particular, we note that the exegesis of the theoretical question of the relation between fluctuation levels and transport dates back to the early 1960s, and that during the past 10 years, a community consensus as to both the ubiquity and importance of *zonal flows* in drift wave turbulence has arisen. Thus, an analysis of the relation between fluctuations and transport in the presence of zonal flows is both relevant and long overdue. Such an analysis is crucial to the long-term goal of relating fluctuations to transport, since direct measurement of turbulent fluxes in the core of relevant plasmas remains too difficult.

The purpose of the paper is to study the effect of random (i.e., broadband) shearing (by zonal flows) on turbulence regulation in a scalar field model. Intuitively, it is clear that shearing becomes ineffective as $\tau_{ZF}\to 0$, since then a shear flow has no time to act on an eddy.[10] In the physically relevant case where τ_{ZF} is larger than τ_c, the correlation time of turbulence, the critical value of the correlation time of the zonal flows, below which the shearing effect is reduced is roughly $\tau_\Omega=\Omega_{rms}^{-1}$, where $\Omega_{rms}=\langle\Omega^2\rangle^{1/2}$ is the rms value of the shear. For $\tau_{ZF}<\tau_\Omega$, the effective shearing rate becomes $\Omega_{eff}=\tau_{eff}^{-1}=\tau_{ZF}\Omega_{rms}^2(<\Omega_{rms})$. It is well known that in the strong shear limit (i.e., $\tau_\Omega/\tau_\eta\to 0$), sheared flows enhance the decorrelation rate of two nearby fluid elements to $\tau_\Delta^{-1}=(Dk^2\Omega^2)^{-1/3}=(\tau_\eta\tau_\Omega^2)^{-1/3}$, above the value $\tau_\eta=(Dk^2)^{-1}$ determined by turbulent scattering alone.[11] Here, D is the effective diffusivity including the effect of nonlinear mixing, and $1/k$ is the characteristic scale of the turbulence. Note that $\tau_\Delta\gg\tau_\Omega$ in the strong shear limit. When the shearing is random, this rate is also reduced to $\tau_D^{-1}=(Dk^2\tau_{ZF}\langle\Omega^2\rangle^2)^{-1/2}=(\tau_\eta\tau_{eff})^{-1/2}$ due to inefficient shearing (i.e., $\tau_\Delta<\tau_D$ for $\tau_{ZF}<\tau_\Delta$). As τ_{ZF} becomes large enough to satisfy the inequality $\tau_{ZF}\gg\tau_D$, the results for the case of steady shear flow are recovered, as zonal flows can then be considered to be steady, albeit with complex spatial structure. Another interesting consequence of random shearing with no net mean shear ($\langle\Omega\rangle=0$) is the substitution of resonance between the flow and turbulence (where a local Doppler shifted frequency vanishes) by a smooth, probabilistic interaction kernel. We recall that resonance underlies irreversibility, and yields a nontrivial transport scaling. The scaling of the flux, however, turns out to be similar to that for the case of a steady linear shear flow, with $\Omega\to\Omega_{rms}$. This reduction is an important benchmark for the theory. In comparison, the amplitude of the scalar field will be shown to increase slightly (with a different scaling) because of longer effective decorrelation time ($\tau_D>\tau_\Delta$).

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Let us consider the transport of scalar field χ by random turbulent flow $\mathbf{v}$ and random zonal flow $\mathbf{U}=U(x,t)\hat{y}$ (with $\langle\mathbf{v}\rangle=\langle U\rangle=0$) in the two dimensional x and y plane,

$$[\partial_t + U(x,t)\partial_y]\chi' = -v_x\partial_x\chi_0 + D(\partial_{xx}+\partial_{yy})\chi', \qquad (1)$$

where χ' and χ_0 are fluctuating and mean parts of χ, and D is the effective diffusivity, including nonlinear interaction. The random turbulent flow is assumed to have characteristic frequency ω_k and correlation time $\tau_c=\gamma_k^{-1}$, while zonal flows have correlation time τ_{ZF}. Since a zonal flow $U(x,t)$ is random, the scalar field flux $\Gamma=\langle\langle\chi' v_x\rangle\rangle$ should be averaged over the ensemble of zonal flows, in addition to that of the turbulent flow $\mathbf{v}$. We shall use angular brackets $\langle\ \rangle$ to denote the average over either one of the two, and use double angular brackets $\langle\langle\ \rangle\rangle$ to denote the average over both. At this point, the alert reader may be somewhat skeptical of the assumption of independent statistics and the probability distribution functions (PDFs) for fluctuations and zonal flows. In this regard, it is useful to think of the fluctuations as complex fields, with amplitude *and* phase. Zonal flows are driven by fluctuation *intensity*, via Reynolds stresses, etc. Thus, the fluctuation statistics have a degree of freedom beyond those of the zonal flow, so we speculate that the interdependence of the two may be regarded as weak. Further study of this interesting point is a topic for future investigations. In the following, we focus on the two interesting limits: (a) when a zonal flow is temporally random on time scales $\tau_{ZF}>\tau_c$, with a fixed linear profile $U(x,t)=x\Omega(t)$, and (b) when the zonal flow is steady, but spatially complex $[U(x,t)=U(x)]$.[12]

We now examine the first case when zonal flows have finite correlation time τ_{ZF} with a linear spatial profile, i.e., $U(x,t)=x\Omega(t)$. The degree to which randomness of zonal flows (finite τ_{ZF}) influences the dynamics depends on whether τ_{ZF} is smaller or larger than other characteristic time scales, such as the shearing time τ_Ω and decorrelation time τ_Δ. As we are interested in the strong shear limit, τ_Δ is taken to be much larger than τ_Ω throughout this paper. Given the uncertainty in τ_{ZF}, physically relevant cases are likely to be (i) $\tau_c<\tau_{ZF}\ll\tau_\Omega\ll\tau_D$ and (ii) $\tau_\Omega\ll\tau_c<\tau_{ZF}\ll\tau_D$. Case (i) corresponds to δ-correlated turbulent (and zonal) flow, where the irreversibility arises mainly from the randomness of the flow while in case (ii), the zonal flow-wave resonance is the main source of irreversibility (in the limit $\tau_{ZF}\to\infty$). It is illuminating that even without complicated analysis, the scaling of flux in case (ii) can be easily obtained, since the long time average of the flux does not depend on the dissipation, rendering it legitimate to take $\tau_{ZF}\to\infty$. Thus, we can simply use the result for a fixed shear flow $\langle\chi' v_x\rangle\propto\delta(\omega_k-x\Omega k_y)$, and then take its average over an ensemble of zonal flows. For simplicity, we perform the latter by assuming Gaussian probability for Ω as $dP[\Omega]=(1/\Omega_{rms})d\Omega e^{-\Omega^2/2\Omega_{rms}^2}$:

$$\langle\langle\chi' v_x\rangle\rangle \propto \frac{1}{xk_y\Omega_{rms}}e^{-\omega_k^2/2x^2k_y^2\Omega_{rms}^2}. \qquad (2)$$

Thus, a sharp resonance $\delta(\omega_k-x\Omega k_y)$ becomes a smooth, probabilistic interaction kernel, making the flux maximal for $\omega_k=\sqrt{2k_y^2x^2\Omega_{rms}^2}$, with its value $\propto\Omega_{rms}^{-1}$. Thus, the flux has a similar scaling with Ω_{rms} as with Ω in the case of a *fixed* shear flow. The same result [Eq. (2)] shall also be obtained below through a more laborious calculation. It is important to note that the same analysis cannot be applied to $\langle\langle\chi'^2\rangle\rangle$, since the long time average of $\langle\chi'^2\rangle$ for a fixed shear is taken over a time longer than τ_D, which is much larger than τ_{ZF}. Thus, a more rigorous analysis is necessary.

To incorporate the shearing effect in Eq. (2), we employ a time-dependent wave number $k_x(t)$ in the direction of the shear (i.e., shearing coordinate), by assuming

$$\chi'(\mathbf{x},t)=\frac{1}{(2\pi)^2}\int d^2k e^{i(k_x(t)x+k_y y)}\tilde{\chi}(k_x(t),k_y,t). \qquad (3)$$

Upon using Eq. (3), and assuming $\partial_t k_x(t)=-k_y\Omega(t)$, Eq. (1) can be easily solved as

$$\tilde{\chi}(k_x(t),k_y,t) = -\partial_x\chi_0\int d^2k_1\int^t dt_1 g(\mathbf{k},t;\mathbf{k}_1,t_1)\times e^{-DQ(t_1)}\tilde{v}_x(k_{1x}(t_1),k_{1y},t_1). \qquad (4)$$

Here, $Q(t_1)=k_y^2(t-t_1)+\int_{t_1}^t dt' k_x^2(t')$, and $g(\mathbf{k},t;\mathbf{k}_1,t_1)=\delta(k_y-k_{1y})\delta[k_x-k_{1x}+k_{1y}\int_{t_1}^t dt_2\Omega(t_2)]$ is the Green's function for the evolution of χ'. From Eq. (4), the flux and mean square amplitude of χ', when averaged over the statistics of turbulent flow v_x, are:

$$\langle\chi' v_x\rangle = -\frac{\partial_x\chi_0}{(2\pi)^2}\int^t dt_1 d^2k_1 e^{-ik_y x\int_{t_1}^t dt'\Omega(t')-DQ(t_1)}\phi(\mathbf{k}_1,t-t_1), \qquad (5)$$

$$\langle\chi'^2\rangle = \frac{(\partial_x\chi_0)^2}{(2\pi)^2}\int^t dt_1 dt_2 d^2k_1 e^{-ik_y x\int_{t_1}^{t_2}dt'\Omega(t')-D[Q(t_1)+Q(t_2)]}\times\phi(\mathbf{k}_1,t_1-t_2). \qquad (6)$$

Here, again $Q(t_i)=k_y^2(t-t_i)+\int_{t_i}^t dt' k_x^2(t')$ for $i=1,2$, and stationary and homogeneous turbulence for v_x has been assumed with $\langle\tilde{v}_x(\mathbf{k}_1,t_1)\tilde{v}_x(\mathbf{k}_2,t_2)\rangle=(2\pi)^2\delta(\mathbf{k}_1+\mathbf{k}_2)\phi(\mathbf{k}_1,t_2-t_1)$.

For case (i), we can take $\phi(\mathbf{k},t_2-t_1)=\tau_c\delta(t_1-t_2)\psi(\mathbf{k})$ since τ_c is the shortest time scale in the system. Then, it is trivial to see that the effect of shearing for the flux vanishes as $\langle\langle\chi' v_x\rangle\rangle=\langle\chi' v_x\rangle=-(\tau_c\partial_x\chi_0/(2\pi)^2)\int d^2k\psi(\mathbf{k})$, which is consistent with the result for a steady shear flow. On the other hand, the effect of dissipation D, enhanced by zonal flow shearing, is critical to determining the amplitude $\langle\langle\chi'^2\rangle\rangle$. This is because of the generation of fine scales in x (or large k_x), and requires the following quantity to be averaged over an ensemble of zonal flows:

$$I_z \equiv \langle e^{-D\int_0^\tau dt' k_x^2(t')}\rangle. \qquad (7)$$

Since the argument of the exponential is quadratic in Ω with nonvanishing mean value $[\partial_t k_x(t)=-k_y\Omega(t)]$, the average can be evaluated for each term by assuming Gaussian statistics for Ω, after expanding the exponential function. Of course, other forms of the probability distribution function should be considered as well. For this average, shearing can be treated as a random walk over τ_D, since the former changes many times, so long as $\tau_{ZF}\ll\tau_D$. For instance, it is reasonable to

TABLE I. Summary of results for zonal flows with finite correlation time τ_{ZF}.

	$\tau_c < \tau_{ZF} < \tau_\Omega \ll \tau_D$	$\tau_\Omega < \tau_c < \tau_{ZF} \ll \tau_D$	$\tau_D \ll \tau_{ZF}$
$\langle\langle \chi' v_x \rangle\rangle$	Ω_{rms}^0	Ω_{rms}^{-1}	Ω_{rms}^{-1}
$\langle\langle \chi'^2 \rangle\rangle$	Ω_{rms}^{-1}	$\Omega_{rms}^{-2} D^{-1/2}$	$\Omega_{rms}^{-5/3} D^{-1/3}$

take $\int_0^t dt_1 \int_0^t dt_2 \langle \Omega(t_1)\Omega(t_2)\rangle \simeq t\tau_{ZF}\langle\Omega^2\rangle$. By using this and focusing on the strong shear limit $Dk_y^2 < \tau_{ZF}\langle\Omega^2\rangle$, we scale the time with $\tau_D = (Dk_y^2 \tau_{ZF}\langle\Omega^2\rangle)^{-1/2}$ in the resulting average. This will reduce the time integral in Eq. (6) to $\int^t dt_1 e^{-DQ(t_1)} = \tau_D \int_0^\infty ds I_z(s) = \mathcal{C}\tau_D$ in the long time limit ($t\to\infty$), yielding a convergent integral $C \equiv \int_0^\infty ds I_z(s)$. Thus, the amplitude becomes

$$\langle\langle \chi'^2 \rangle\rangle = \frac{(\partial_x \chi_0)^2 \tau_c \tau_D \mathcal{C}}{(2\pi)^2} \int d^2k \phi(\mathbf{k}) \propto [Dk_y^2 \tau_{ZF}\langle\Omega^2\rangle]^{-1/2}. \quad (8)$$

As compared with the amplitude $\langle \chi'^2 \rangle \propto \tau_\Delta$ ($>\tau_D$ for $\tau_{ZF} < \tau_D$) for the case of coherent shearing, random shearing clearly yields a larger scalar fluctuation amplitude. Equation (8) also explicitly shows that the amplitude increases as τ_{ZF} decreases, as it is expected to. The upper bound on the amplitude is, however, given by $\tau_D = (Dk_y^2\langle\Omega^2\rangle)^{-1}$ in order to satisfy the assumption of the strong shear limit $Dk_y^2 < \tau_{ZF}\langle\Omega^2\rangle$. Note that for a steady flow with constant Ω, the argument of the exponential in Eq. (7) becomes proportional to τ^3, resulting in a characteristic time scale τ_Δ. When $\tau_{ZF} \gg \tau_D$, the zonal flow can be treated as steady, so the amplitude is proportional to τ_Δ. These results are summarized in Table I.

For case (ii), we assume a Lorentzian frequency spectrum for ϕ as $\phi(\mathbf{k}, t_2 - t_1) = \int d\omega \psi(\mathbf{k}) e^{-i\omega(t_2-t_1)} \gamma_k / [(\omega - \omega_k)^2 + \gamma_k^2]$. Then, after performing the ω integral, the ensemble average of the flux becomes

$$\langle\langle \chi' v_x \rangle\rangle = -\frac{\partial_x \chi_0}{(2\pi)^2} \int^t d\tau d^2k \psi(\mathbf{k}) \langle e^{-ik_y x \int_0^\tau dt' \Omega(t')} \rangle, \quad (9)$$

where $\tau = t - t_1$. By using Gaussian statistics for Ω, the ensemble average over zonal flows can be approximated as $\langle e^{-ik_y x \int_0^\tau dt' \Omega(t')}\rangle = e^{-((k_y x)^2/2)\int_0^\tau dt' dt'' \langle \Omega(t')\Omega(t'')\rangle} \simeq e^{-((k_y x)^2/2)\tau^2 \langle\Omega^2\rangle}$. Note that zonal flow shearing was taken to be coherent over the time interval $t \in [0, \tau]$, since the effective shearing time τ_Ω is shorter than τ_{ZF} and also since turbulence varies on the fast time scale γ_k^{-1}, thus giving a major contribution to the τ integral from small τ. The remaining τ integral then gives (by assuming $\omega_k > \gamma_k$)

$$\langle\langle \chi' v_x \rangle\rangle \simeq -\frac{\partial_x \chi_0}{2(2\pi)^{2/3}} \int d^2k \psi(\mathbf{k}) \frac{e^{-\omega_k^2/(2k_y^2 x^2 \langle\Omega^2\rangle)}}{(k_y^2 x^2 \langle\Omega^2\rangle)^{1/2}}. \quad (10)$$

It is amusing to see that the scaling in Eq. (10) is exactly what Eq. (2) predicted. The latter was obtained by taking an average of the existing result for a steady shear flow. We again note that the flux in Eq. (10) takes its maximum value for $\omega_k = \sqrt{2k_y x^2 \langle\Omega^2\rangle}$, (which replaces the resonance condition), and is $\sim \Omega_{rms}^{-1}$.

In order to compute the amplitude for case (ii), we note that the effect of dissipation enters on a long time scale (τ_D), compared to other time scales. Thus, we envision taking average of zonal flows over two different time scales T_1 and T_2, where $\tau_{ZF}, \tau_c < T_1 < \tau_D$ and $T_2 > \tau_D$. Thus, we can first ignore the dissipation (i.e., $D=0$) and take average over T_1, and then take average over T_2 with $D \neq 0$. Then, analyses similar to those given above yield

$$\langle \chi'^2 \rangle = \frac{(\partial_x \chi_0)^2}{(2\pi)^{2/3}} \int d^2k \psi(\mathbf{k}) \frac{\tau_D \mathcal{C} e^{-\omega_k^2/(2k_y^2 x^2 \langle\Omega^2\rangle)}}{(k_y^2 x^2 \langle\Omega^2\rangle)^{1/2}} \propto \frac{1}{\langle\Omega^2\rangle (Dk_y^2 \tau_{ZF})^{1/2}}, \quad (11)$$

where again $\mathcal{C} \equiv \int^\infty ds I_z(s)$ is a convergent integral. Equation (11) reveals that the amplitude in the case of random shear is enhanced due to the inefficiency of turbulence regulation. This is due to the shear's random character, as compared to a steady shear, where $\langle \chi'^2 \rangle \propto \tau_\Delta / \Omega$. For instance, the amplitude increases as τ_{ZF} becomes small, as expected. The upper limit on the amplitude is, however, again given by $\tau_D = (Dk_y^2 \langle\Omega^2\rangle)^{-1}$. On the other hand, as τ_{ZF} becomes larger than τ_D, the parameter τ_D in Eq. (11) should be replaced by τ_Δ, since zonal flows can then be treated as steady flows.

We will now show that in the case of a steady zonal flow with complex spatial dependence, the results are similar to those in the case of linear shear flow, provided that Ω is replaced by the rms shearing rate $\Omega_{rms} = \langle (\partial_x U)^2 \rangle^{1/2}$, in agreement with Ref. 12. Since it is plausible that l_{ZF}, the correlation length of zonal flows, can be comparable to l_c, that of the turbulence, we can no longer Fourier decompose χ' in x. Therefore, we Fourier transform only in y and introduce a phase function $g(x,t)$ as

$$\chi'(\mathbf{x}, t) = \frac{1}{2\pi} \int dk_y e^{ik_y y + g(x,t)} \tilde{\chi}(k_y, x, t). \quad (12)$$

By assuming that $|\partial_x g / g| \gg |\partial_x \tilde{\chi} / \tilde{\chi}|$, and by using the usual Fourier transform for v_x as $v_x(\mathbf{x}, t) = 1/2\pi \int dk_y e^{ik_y y} \tilde{v}(k_y, x, t)$, Eq. (1) can easily be solved for $\tilde{\chi}$ as

$$\tilde{\chi}(x, k_y, t) = -\partial_x \chi_0 \int^t dt_1 e^{-D[\bar{Q}(k,t) - \bar{Q}(k,t_1)] - g(x,t_1)} \tilde{v}_x(k_y, x, t_1). \quad (13)$$

Here, $\bar{Q}(k,t) = k_y^2 t + (k_y U')^2 t^3/3 + ik_y U'' t^2/2$ with $U' = \partial_x U$ and $U'' = \partial_{xx} U$. By exploiting the conditions of "steady and homogeneous turbulence" with the correlation function $\langle \tilde{v}_x(x, k_y, t_1) \tilde{v}_x(x, k_y', t_2) \rangle = (2\pi)\delta(k_y + k_y')\phi(k_y, t_2 - t_1)$, we obtain,

$$\langle \chi' v_x \rangle = -\frac{\partial_x \chi_0}{(2\pi)} \int^t dt_1 dk_y e^{-ik_y U(t-t_1) - D[\bar{Q}(k,t) - \bar{Q}(k,t_1)]} \phi(k_y, t - t_1), \quad (14)$$

$$\langle \chi'^2 \rangle = \frac{(\partial_x \chi_0)^2}{(2\pi)} \int^t dt_1 dt_2 dk_y \times e^{-ik_y U(t_2-t_1) - D[\bar{Q}(k,t)+\bar{Q}(-k,t)-\bar{Q}(k,t_1)-\bar{Q}(-k,t_2)]} \phi(k_y, t_2 - t_1). \tag{15}$$

First, it is easy to see that for a δ correlated flow v_x (i.e., $\tau_c \ll \tau_\Omega$), the flux is independent of U. However, the computation of $\langle\langle \chi'^2 \rangle\rangle$ requires an average (over zonal flows) like $\langle e^{-2D(k_y U')^2 (t^3 - t_1^3)/3} \rangle$. Since the scaling of the amplitude with the rms shear is of greatest interest, this computation can be done by following an analysis similar to that done previously to obtain Eq. (8), and then scaling the time with $(Dk_y^2 \langle U'^2 \rangle)^{1/3}$. The result is

$$\langle \chi'^2 \rangle \propto \frac{(\partial_x \chi_0)^2 \tau_c}{(2\pi)} \int dk_y \psi(k_y) \frac{1}{[Dk_y^2 \langle U'^2 \rangle]^{1/3}}. \tag{16}$$

Therefore, the scaling of the amplitude with rms shear and D are the same as those in the case of a linear shear flow, provided that $\langle U'^2 \rangle^{1/2}$ replaces constant Ω.

For $\tau_\Omega < \tau_c < \tau_D$, we again use a Lorentzian frequency spectrum for v_x. After straight-forward algebra using Gaussian statistics for U, U', and U'', and $\langle UU' \rangle = \langle U'U'' \rangle = 0$, we can obtain the following scalings:

$$\langle \chi' v_x \rangle \simeq -\frac{\partial_x \chi_0}{(2\pi)} \int dk_y \psi(k_y) \frac{e^{-\omega_k^2/(2k_y^2 \langle U^2 \rangle)}}{|2k_y \langle U^2 \rangle^{1/2}|}, \tag{17}$$

$$\langle \chi'^2 \rangle \propto \frac{(\partial_x \chi_0)^2}{(2\pi)} \int dk_y \psi(k_y) \frac{e^{-\omega_k^2/(2k_y^2 \langle U^2 \rangle)}}{|2k_y \langle U^2 \rangle^{1/2}| [Dk_y^2 \langle \Omega^2 \rangle]^{1/3}}. \tag{18}$$

Due to the spatial randomness of the zonal flow pattern, resonance between zonal flow and turbulence is smoothed out, with the maximum flux occurring when $\omega_k = \sqrt{2k_y^2 \langle U^2 \rangle}$, as in temporally random case [see Eq. (10)]. Therefore, the scalings of the amplitude with shear and D are basically the same as those for the case of a linear shear flow, provided that $\langle U'^2 \rangle^{1/2}$ is replaced by Ω. We note that the curvature effect U'' does not appear in the final amplitude, since a strong shear limit $D^2 k_y^2 \langle U''^2 \rangle / [Dk_y^2 \langle U'^2 \rangle]^{4/3} \sim [(Dk_y^2)^2 / \langle U'^2 \rangle]^{1/3} (l_c / l_{ZF})^2 \ll 1$ was assumed.

In summary, we have shown that the effect of random shearing of zonal flows with finite autocorrelation time on transport and fluctuation levels of scalar fields crucially depends on the zonal flow pattern and correlation time τ_{ZF}. For spatially random zonal flows $U(x,t) = U(x)$ ($l_{ZF} \gtrsim l_c$) with infinite memory time, the same scalings of flux and amplitude of scalar fields with $\langle U'^2 \rangle^{1/2}$ are obtained as in the case of a steady linear shear flow (with $\langle U'^2 \rangle^{1/2}$ replacing Ω).[12] More interesting results were found for zonal flows with finite correlation time τ_{ZF} [i.e., $U(x,t) = x\Omega(t)$] (see Table I). For $\tau_c < \tau_\Omega \ll \tau_{ZF} \ll \tau_D$, the flux becomes independent of shear to leading order, while $\langle\langle \chi'^2 \rangle\rangle \propto \Omega_{rms}^{-1}$. In the physically more interesting case where $\tau_\Omega \ll \tau_c < \tau_{ZF} \ll \tau_D$, $\langle\langle \chi' v_x \rangle\rangle \propto \Omega_{rms}^{-1}$ while $\langle\langle \chi'^2 \rangle\rangle \propto \Omega_{rms}^{-2} D^{-1/2}$. The scaling of the latter, which is different from $\langle \chi'^2 \rangle \propto \Omega^{-5/3} D^{-1/3}$ in the case of coherent shearing Ω, is a result of the longer effective decorrelation time of fluid elements $\tau_D > \tau_\Delta$ induced by finite zonal flow autocorrelation time τ_{ZF}. As τ_{ZF} exceeds τ_D, zonal flows can be considered to be steady in time, thus recovering previous results.

The results of this paper highlight the *great importance of the determination of both the frequency spectrum* (in particular, the correlation time τ_{ZF}) *and PDF of zonal flows*, in both simulations and physical experiments. In particular, we have assumed a Gaussian PDF of zonal flows throughout this paper, but there are likely cases for which the PDF of zonal flows is exponential or even power law. Experimentally, a useful estimate on τ_{ZF} can be obtained from constructing the average two time correlation function of a zonal flow $V_E \hat{y}$, i.e., $\tau_{ZF} = \int_0^\infty dt \langle V_E(\tau) V_E(\tau + t) \rangle / \langle V_E(\tau)^2 \rangle$, or from the width of the $m=0$ frequency spectrum. We finally note that the methodology and approach of this work is relevant to geodesic acoustic modes, but that the detailed analysis would require the formulation of a toroidal model and the inclusion of a broader range of zonal flow frequencies.

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